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Interim Report on (NASA ~~Contract~~ Grant NsG-350-63)
Department of Mechanical Engineering
The University of Rochester, Rochester, N.Y.

762 1006 U, # (NASA CR 51811)

The purpose of this contract is to investigate the [feasibility of a travelling wave device of rotationally symmetric geometry for propulsion]. The program has been approached both analytically and experimentally and is described in some detail below; following which is a discussion of proposed future study

[1962]

N65 81581

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INTRODUCTION

The purpose of the annular travelling wave propulsion channel is to accelerate a cold plasma from a low inlet velocity (for example 10 to 100 meters per second) to an exit velocity in the range of 15 to 20 thousand meters per second A high frequency magnetic field transfers energy to the ionized particles. These ions are rapidly accelerated and experience charge exchange collisions with the neutral atoms that are present in the medium. After an encounter between a fast ion and a slow neutral particle, the high speed ion captures an electron, becomes a fast neutral, and leaves the tube without further incident. The newly created slow ion is then under the influence of the magnetic field and the process is repeated.

Although three distinctly different species are present, ions, slow neutrals, and fast neutrals we first consider a model which represents the average properties of the plasma. In our initial study the following problems are examined:

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1) A theoretical study of a one fluid continuum travelling wave device.

2) A theoretical study of a multicomponent fluid model for the same geometry.

3) The electromagnetic boundary valued problem to determine the necessary ampere turns for the desired magnetic field.

These are the major theoretical points which will be discussed in this report. In addition, several areas have been given considerable attention in the experimental phase of the project. Among these were:

1. Design and construction of penetrations into the vacuum pumping facility into which the device will operate.

2. Design and construction of a travelling wave tube and winding; selection of an economical core material for the external magnetic circuits and construction of same.

3. Finding and securing a satisfactory multiphase a.c. machine for powering the device.

4. Design and construction of a glow discharge chamber and appropriate power supply to be used as a possible cold plasma source.

5. Design of probes for diagnostic purposes with regard to a.c. magnetic flux.

The above experimental work has occupied considerable time and capital expense but it is felt that it is necessary to verify the several assumptions involved in the analysis, for example, the importance of charge exchange phenomena.

A Theoretical Study of a One Fluid Continuum Travelling Wave Tube

We begin our analysis with the basic magnetohydrodynamic equations that characterize the flow. It is assumed that the magnetic Reynolds number is very small and therefore the flow does not distort the imposed magnetic field.

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \qquad (1)$$

Momentum

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = (\bar{J} \times \bar{B})_x \qquad (2)$$

Ohms Law

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \qquad (3)$$

Where:

ρ = mass density Kgm/meter^3

u = plasma velocity in the axial direction meters/sec

x = axial coordinate-meters

t = time-seconds

J = current, amperes

B = magnetic field-Webers/meter^2

E = electric field-volts/meter

σ = electrical conductivity-mho's/meter

After combining equation (2) and (3), the two remaining expressions are non-dimensionalized and show that the variables ρ and u can be separated if one assumes that each of them may be represented by the series expansion as in reference 1.

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$$q(x_1 t) = \sum_{M=\infty}^{\infty} \sum_{K=V}^{\infty} \epsilon^{|M|+K} g_{MK}(v) \, \epsilon^{2iNC} \qquad (4)$$

Where: g = general property (ρ or u)

ϵ = an arbitrary expansion parameter that will be introduced into the definition of the $\overline{J} \times \overline{B}$ force.

The complete development indicates that the coupling between the magnetic field and the plasma is directly proportional to the conductivity of the plasma, and the square of the maximum amplitude of the field, and inversely proportional to the initial momentum flux.

We will now present a detailed solution of the problem and then discuss the effect of varying some of the aforementioned parameters.

Figure 1 shows the basic geometry and indicates that an elemental volume of plasma accelerated by the magnetic field, moves to the right. In reality, the volume will follow some helical path in the annulus; however, for a one-dimensional model the motion is only axial.

The current J induced in the plasma is a function of the relative velocity between the plasma and the field. These two velocities are related by the equation:

$$v_s = v_g - v_B .$$

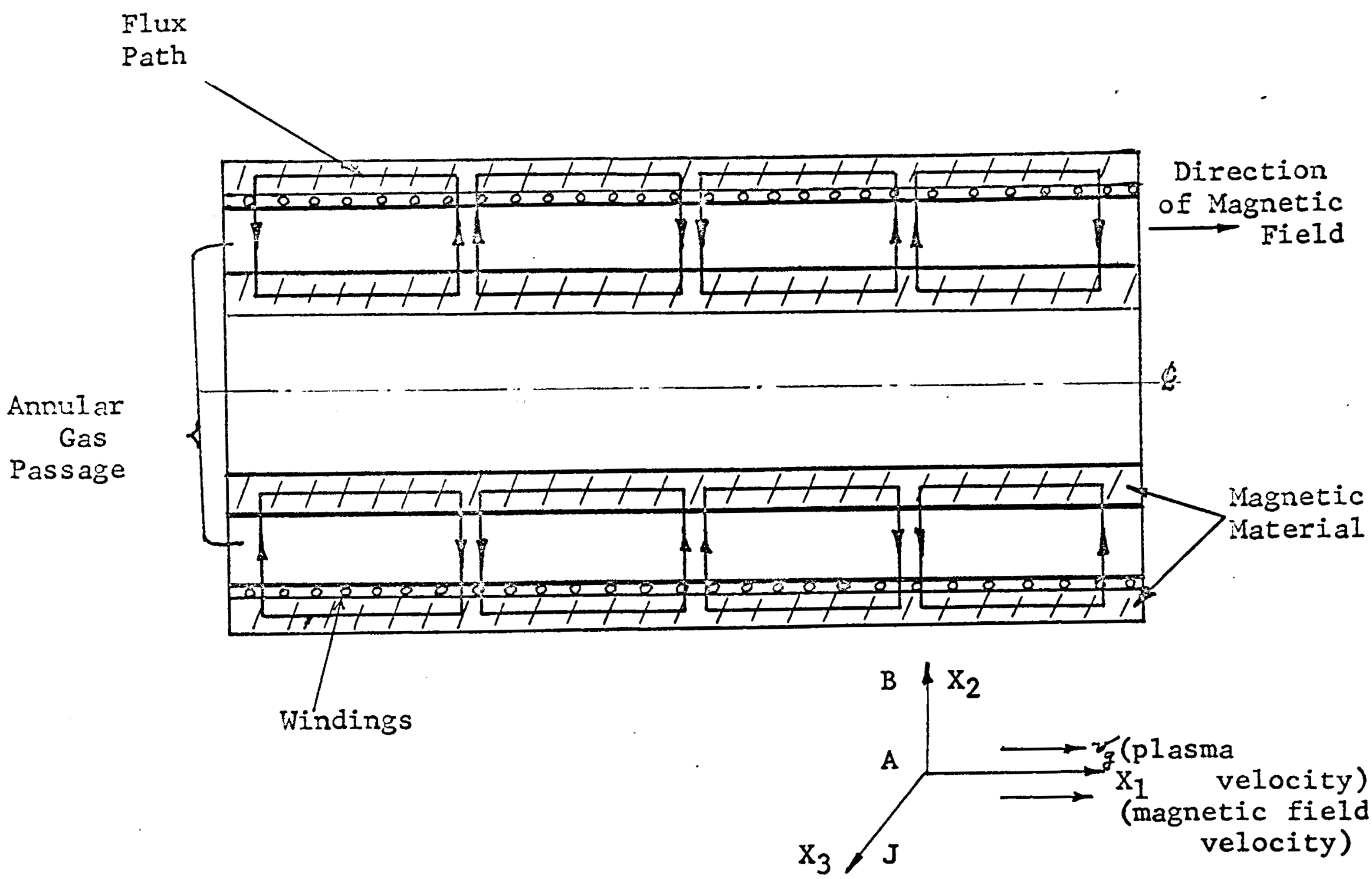


Fig. 1

and since the fluid velocity can never equal the field, the relative velocity vector is always in the negative x direction. When travelling with the fluid, the plasma only experiences a magnetic field; therefore, Ohms Law is written as

$$\vec{J} = \sigma(\vec{v} \times \vec{B}) = -\sigma v_s B \vec{X}_3 \tag{5}$$

It is assumed that the field is a periodic function of the form

$$B = B_o \cos\left(\frac{2\pi x}{\lambda} - \omega t\right) \tag{6}$$

Where:

λ = wavelength-meters

ω = radians/sec.

and $v_B = \omega \lambda$

Combining equations (5) and (6) with (2) the result is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\sigma v_s B_o^2 \cos^2\left[\frac{2\pi x}{\lambda} - \omega t\right] \qquad (7)$$

At this point it is appropriate to nondimensionalize equations (1) and (7) by using the following definitions:

$t' = \omega t$

$x' = \frac{2\pi x}{\lambda}$

$u = v_q = v_B u'$

$\rho = \rho_o \rho'$ (ρ_o = mass density at x=0)

The primes indicate the dimensionless quantities, and we now have

<u>Cont.</u>

$$\omega \rho_o \left[\frac{\partial \rho'}{\partial t'} + \frac{\partial}{\partial x'}(\rho' u')\right] = 0 \qquad (8)$$

<u>Mom.</u>

$$\rho'\left[\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'}\right] = \gamma(1-u')\cos^2(x'-t') \qquad (9)$$

and $\gamma = \frac{B_o^2 \sigma}{\omega \rho_o}$ (interaction parameter).

The solution of these coupled equations is made possible by the introduction of a parameter ϵ in the forcing function of the momentum relationship. Hence, we let

$$\cos^2(x'-t') = \frac{1}{2} + \frac{\epsilon}{4}\left(e^{2i(x'-t')} + e^{-2i(x'-t')}\right) \tag{10}$$

which reduces to the actual definition of the cosine squared as $\epsilon \rightarrow 1$.

This fact plus the assumed form of the solution outlined in equation (4) produces the final structure of the momentum and continuity equations. For convenience we will omit the prime notation.

<u>Cont.</u>

$$\sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} 2in\, \epsilon^{|n|+k} \rho_{nk}(x)\, e^{2int} + \sum_{\substack{n=-\infty \\ m=-\infty}}^{\infty} \sum_{\substack{k=0 \\ p=0}}^{\infty} \left\{ \epsilon^{|n|+|m|+k+p} \frac{d}{dx}\left[u_{nk}(x)\, \rho_{mp}(x)\right] e^{2i(n+m)t} \right\} = 0 \tag{11}$$

<u>Mom.</u>

$$\sum_{\substack{n=-\infty \\ m=-\infty}}^{\infty} \sum_{\substack{k=0 \\ p=0}}^{\infty} 2in\, \epsilon^{|n|+|m|+k+p} \rho_{nk}(x)\, u_{mp}(x)\, e^{2imt} + \sum_{\substack{n=-\infty \\ m=-\infty}}^{\infty} \sum_{\substack{k=0 \\ p=0}}^{\infty} \epsilon^{|n|+|m|+|r|+k+p+s} \rho_{nk}(x)\, u_{np}(x) \frac{d}{dx}\left[u_{rs}\right] e^{2i(n+m+r)t} \tag{12}$$

$$= \gamma \left[1 - \sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} \epsilon^{|n|+k} u_{nk}(x)\, e^{2int}\right]\left[\frac{1}{2} + \epsilon\, \frac{e^{2i(x-t)} + e^{-2i(x-t)}}{4}\right]$$

As a first approximation we determine $u_{\sigma o}(x)$ and $\rho_{\sigma o}(x)$. In so doing we expand equations (10) and (12) in powers of ϵ and set each of the coefficients of ϵ equal to zero. After following the procedure, and uncoupling the equations we find that

$$u_{\sigma o} = 1 + \frac{\beta}{\exp\left[\frac{r}{2K} x\right]}$$

and

$$\rho_{\sigma o} = \frac{K}{u_{\sigma o}} = K \frac{\exp\left[\frac{r}{2K} x\right]}{\beta + \exp\left[\frac{r}{2K} x\right]}$$

where

$$\beta = \left.\frac{v_g}{v_B}\right|_{x=0} - 1 = \frac{v_{g_0}}{v_B} - 1$$

and

$$K = \rho_{\sigma o} \cdot u_{\sigma o} = (1) \left.\frac{v_g}{v_B}\right|_{V=0} = \frac{v_{g_0}}{v_B}.$$

Figure 2 illustrates the degree of coupling between the magnetic field and the plasma, and in the final analysis

$$\frac{\alpha'}{2K} = \alpha = \frac{B_0^2 \sigma v_{g_0}}{\omega \rho_0 v_{g_0}} \cdot \frac{v_B}{(2) v_{g_0}} = \frac{B_0^2 \sigma}{4\pi \rho_0 v_{g_0}}$$

which indicates the coupling is independent of the frequency.

It is desirable to have as large a value of α as possible. The initial density must be large enough so that we can measure a change in its velocity yet low enough to assure the reasonableness

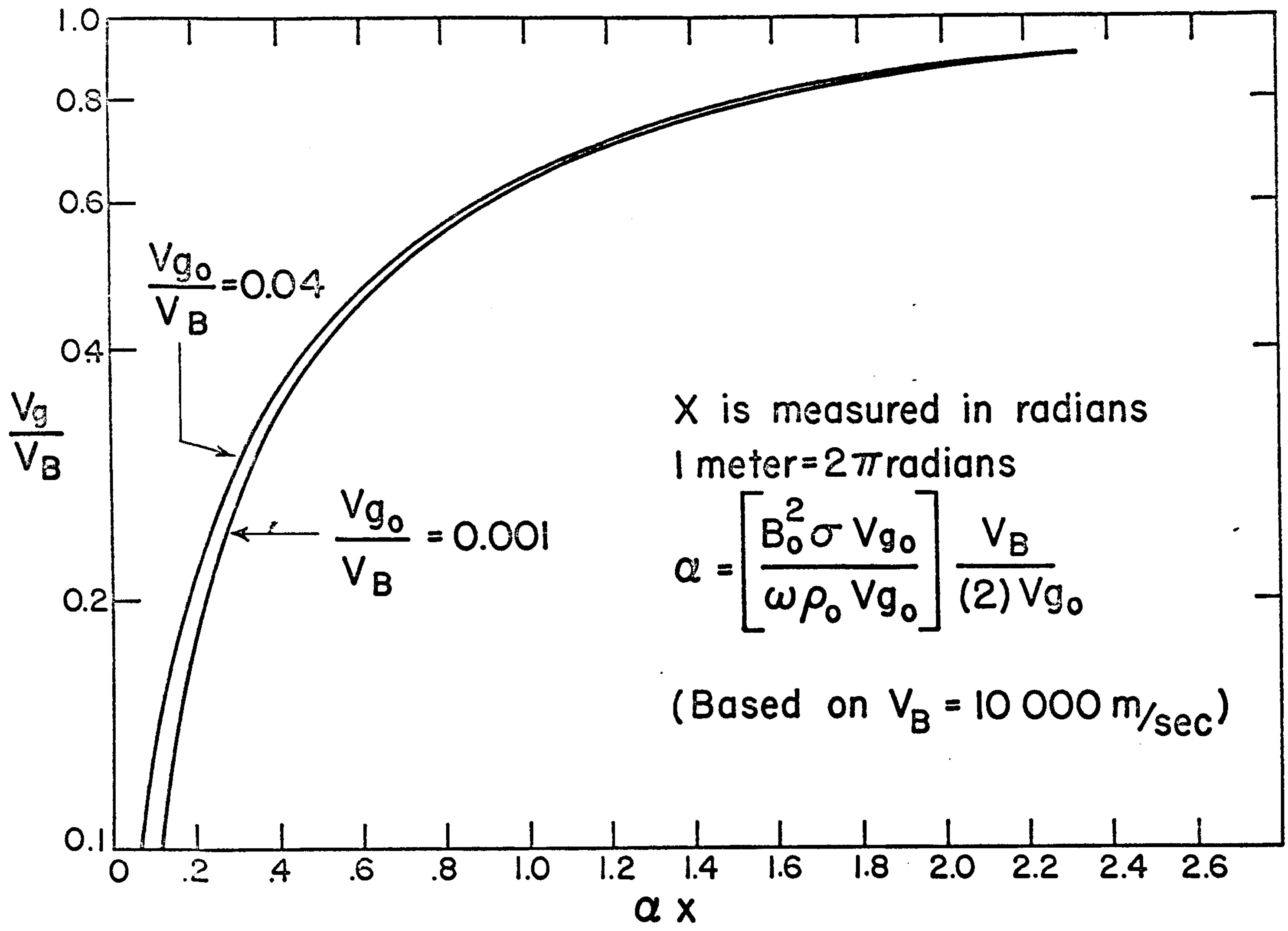


FIG 2. PLOT of $\frac{Vg}{V_{B_o}}$ vs αx

of the model of charge exchange interaction. The conductivity will be influenced by the efficiency of a.c. field excitation to produce ions initially and the plasma source. Figure 3 shows that the conductivity of xenon rises very rapidly with temperature change but is relatively insensitive to variations in pressure. In another section we will discuss the boundary valued problem effecting the average value of the magnetic field as a function of ampere turns.

The Multicomponent Fluid Model

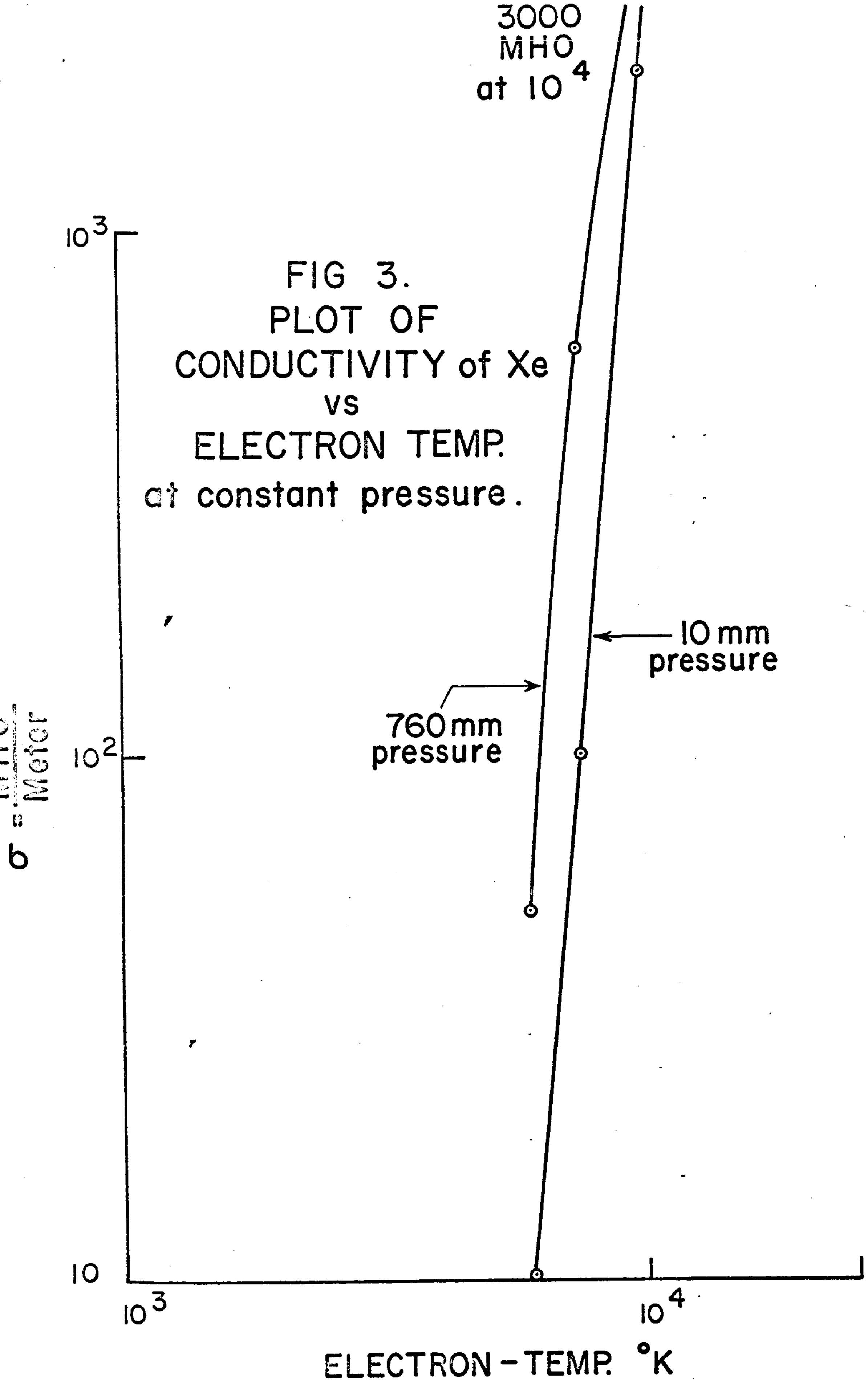
The multicomponent fluid model is solved in a similar manner. The basic system of equations is different, however, to include the effect of charge exchange in both species generation and momentum transfer. Three fluids are assumed in this section, they are:

1) Ion-electron gas designated by subscript i.
2) Fast neutral gas designated by subscript f.
3) Slow neutral gas designated by subscript s.

The basic equations are given below for the one-dimensional model.

$$\frac{\partial \rho_k}{\partial t} + \frac{\partial}{\partial x}(\rho_k u_k) = \dot{\omega}_k \qquad \text{Continuity} \qquad (13)$$

$$\sum_k \dot{\omega}_k = 0 \qquad k = i, f, s \qquad \text{Species generation} \qquad (14)$$


3000
MHO
at 10^4
FIG 3.
PLOT OF
CONDUCTIVITY of Xe
vs
ELECTRON TEMP.
at constant pressure.
10 mm
pressure
760mm
pressure
10^3
10^2
10
$\sigma = \frac{MHO}{Meter}$
10^3
10^4
ELECTRON-TEMP. °K

$$\rho_k\left(\frac{\partial u_k}{\partial t}+u_k\frac{\partial u_k}{\partial x}\right)=\dot{\omega}_k\Delta u_k+F \qquad \text{Momentum} \qquad (15)$$

where k may be i, f or s for each of the fluids

ρ_k & u_k have the same meaning as before

$\dot{\omega}_k$ is the generation of the species in the model

F is the external force and body force. In addition, species energy equations may be used. However, as a first solution the effects of pressure gradient are neglected, transport coefficients are assumed constant and hence the energy expressions may be omitted.

With the above stated restrictions, the system of equations to be solved are:

$$\frac{\partial \rho_i}{\partial t}+\frac{\partial}{\partial x}(\rho_i u_i)=0 \qquad (16)$$

which assumes the ion-electron population to remain the same.

$$\frac{\partial \rho_f}{\partial t}+\frac{\partial}{\partial x}(\rho_f u_f)=-\dot{\omega}_s \qquad (17)$$

$$\frac{\partial \rho_s}{\partial t}+\frac{\partial}{\partial x}(\rho_s u_s)=\dot{\omega}_s \qquad (18)$$

where equation (14) has been involved together with the assumption in (16).

$$\rho_i\left(\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}\right)=\dot{\omega}_s(u_i-u_s)+\sigma(u_c-u_i)B^2 \qquad (19)$$

where u_c = field velocity

$$\rho_f\left(\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial x}\right)=-\dot{\omega}_s u_i \qquad (20)$$

$$\rho_s \left(\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} \right) = + \dot{\omega}_s u_s \tag{21}$$

Before proceeding to the solution, the species generation term $\dot{\omega}_s$ must be given significance. If η = collision frequency between ions and slow neutrals, one has

$$\eta = n_i n_s \sigma_{is} (u_i - u_s) \tag{22}$$

where n_i = number density of ions

n_s = number density of slow neutrals

σ_{is} = charge exchange cross section between ions and slow neutrals. Since the mass of ions, slow neutrals and fast neutrals are essentially the same, m,

$$\dot{\omega}_s = -\eta m = -\frac{\sigma_{is}}{m} \rho_i \rho_s (u_i - u_s) \tag{23}$$

wherein the loss of slow neutral mass density is equal to the number of collisions per unit time per unit volume times particle mass.

Again, the expansion (4) is used to reduce the equations to solvable form. The first term solution for the various components of density and velocity are

$$\begin{aligned} u_{s/oo} &= Ax + \frac{\bar{u}}{u_c} \\ \rho_{s/oo} &= k \\ u_{i/oo} &= \beta + \left(\frac{\bar{u}}{u_c} - \beta\right) e^{-Jx} \\ \rho_{i/oo} &= \frac{(1-k)\,\bar{u}/u_c}{\beta + (\bar{u}/u_c - \beta) e^{-Jx}} \end{aligned} \tag{24}$$

where $\bar{u}/u_c$ = dimensionless inlet velocity

k = fraction of inlet gas that is neutral

$A = -\frac{\sigma_{is}}{m}\bar{\rho}\lambda(1-k)\frac{\bar{u}}{u_c}$ where $\bar{\rho}$ is the density of the incoming fluid, λ is the field wavelength.

$J = \frac{\sigma_{is}}{m}\bar{\rho}\lambda k + \frac{\gamma\lambda}{\bar{\rho}\bar{u}(1-k)}$ where γ is the interaction parameter described after equation (9)

$\beta = \left(\frac{\gamma\lambda}{\bar{\rho}\bar{u}(1-k)}\right)/J$ and represents the maximum velocity which the ions can reach as a fraction of the field velocity — approaches 1.

The Charge Exchange Model

Some brief analysis of the charge exchange model is required to indicate its validity as an important consideration in the analysis. From reference (2) and figure (4) it is noted that the charge transfer cross-section decreases to some asymptotic value as the incident ion energy increases approaching the value given by Sena in reference (3). $Q(\text{cm}^{-1}) = 1.88 \times 10^4/u_i^2$ where u_i is the ionization potential in the equation in electron volts. In reference (4) page 7, the elastic effective collision cross-section is reproduced from a work by Brode in 1933. These two results are shown in figure 4, for the rare gases and show very dramatically the effectiveness of charge exchange collisions for the processes previously described. Due to easier ionization, Xenon appears to be a logical gas. However, it is hoped to use both Argon and Xenon as working gases in the experimental program.

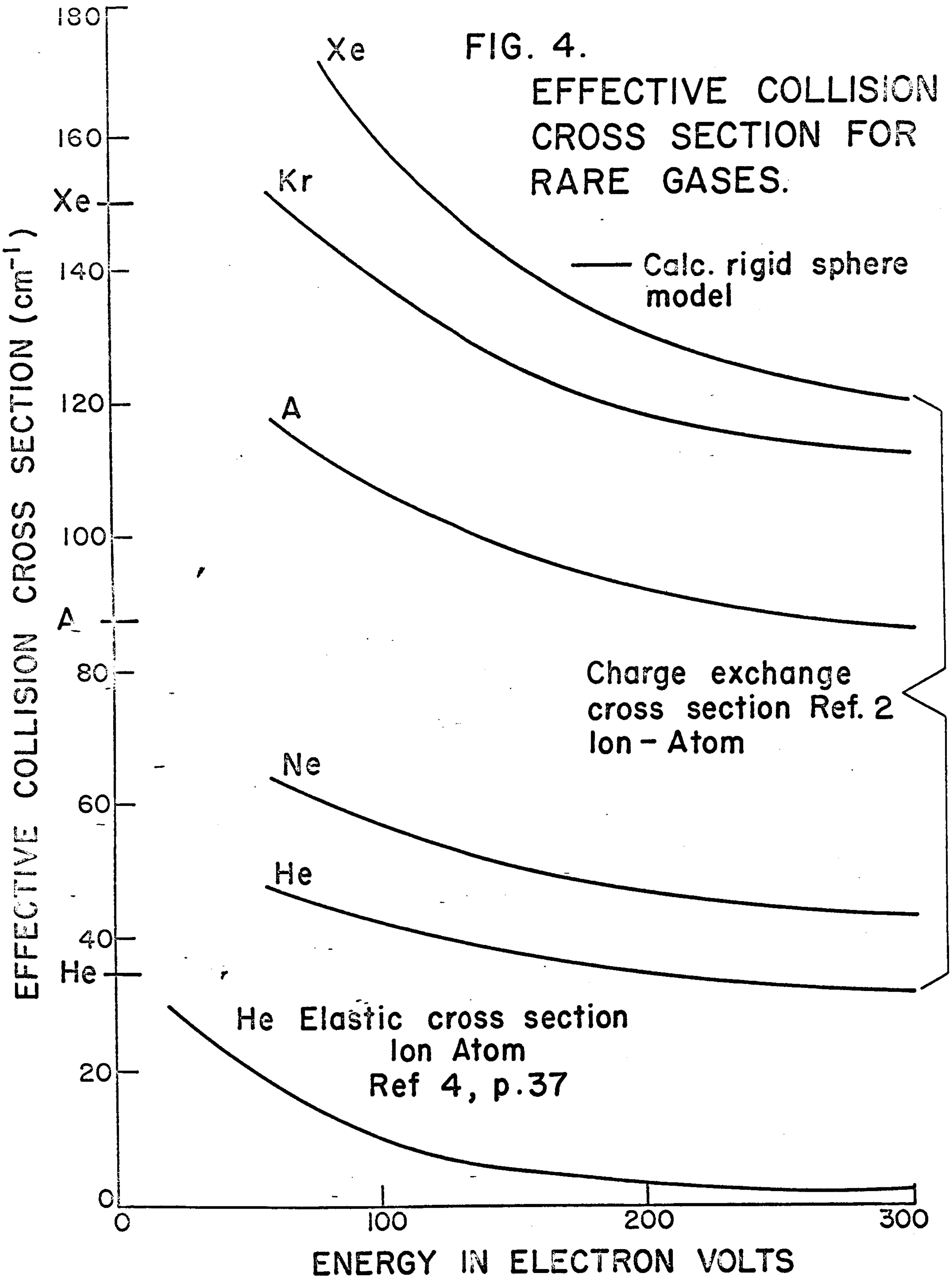


FIG. 4.
EFFECTIVE COLLISION
CROSS SECTION FOR
RARE GASES.
Calc. rigid sphere
model
Charge exchange
cross section Ref. 2
Ion - Atom
He Elastic cross section
Ion Atom
Ref 4, p.37
Xe
Kr
A
Ne
He
EFFECTIVE COLLISION CROSS SECTION (cm^{-1})
180
160
Xe
140
120
100
A
80
60
40
He
20
0
0
100
200
300
ENERGY IN ELECTRON VOLTS

Electromagnetic Boundary Value Problem

Another problem which needed investigation both analytically and experimentally was the field **form** which was assumed in the solutions to be a travelling cosine wave. Before delving into the details of the solution, let us discuss the underlying geometry and rational assumptions that effect the problem. In Figure 5a,

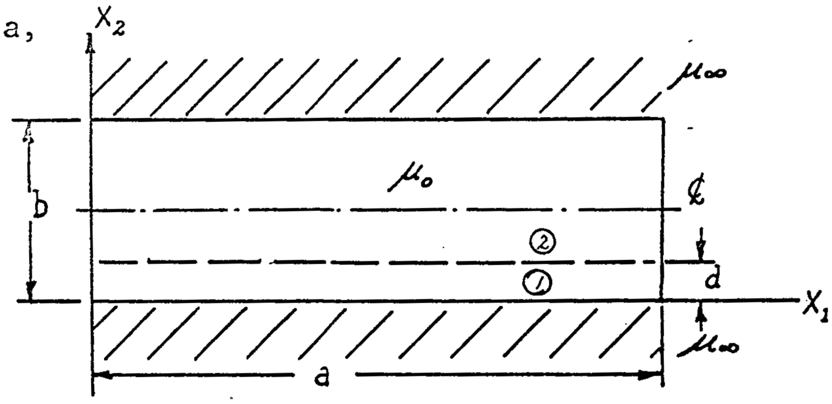


Figure 5a

a = the length of one phase winding

b = the distance from the core to the outer shell

d = the location of the centerline of the windings

℄ = the centerline of the annular region of plasma flow

μ_0 = permeability of free space

μ_∞ = relative permeability of the core material which for analytical purposes will be assumed infinite even though its value is 1200 at a frequency of 20 k c.

The model is two dimensional with dimension a $\approx$ 10b and d $\approx$ 1/7b. One can assert that no current sheet can be present at the interface between the plasma and the magnetic material, because of very low eddy current losses in the steel. Losses range in the neighborhood of 2 watts per pound of core material. From these considerations, the vector potential and associated electromagnatic boundary conditions are

$$\nabla^2 A_z = -\mu I_m \delta(y-d) \tag{25}$$

$$n \cdot (B_\infty - B_o) = 0 \tag{26}$$

$$n \times (H_\infty - H_o) = n \times \left(\frac{B_\infty}{\mu_\infty} - \frac{B_o}{\mu_o}\right) = 0 \tag{27}$$

Where I_m = Maximum value of current

Therefore, we deduce that the tangential component of B at ($x_2 = 0, b$) is zero and the normal component is continuous across these planes. To complete the definition of our problem we set $A_z = 0$ at ($x_1 = 0, a$). The boundary conditions are depicted in 5b

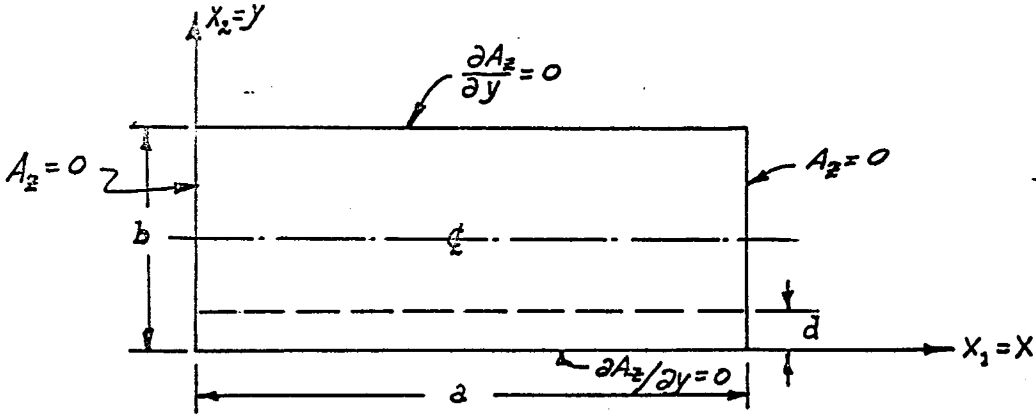


Figure 5b $\bar{B} = \nabla \times \bar{A} = \frac{\partial A_z}{\partial y}\bar{x}_1 - \frac{\partial A_z}{\partial x}\bar{x}_2$

where region (1) is $0 \leq y \leq d$ and region (2) values of $d \leq y \leq b$.

The form of the forcing function in equation (1) implies that at any instant the current is all wires of any phase have the same amplitude. We use standard techniques to solve this problem.

Employing the Green's Function proceed in the following manner;

1) solve the homogeneous equation subject to the boundary conditions of the inhomogeneous equation. We can separate the variables by assuming a solution of the form $A_z = X(x)\,Y(y)$

2) the solution in region one and two must be continuous at y=d

3) the discontinuity in the first derivative is defined by

$$\left.\frac{\partial A_z(1)}{\partial y}\right|_d - \left.\frac{\partial A_z(2)}{\partial y}\right|_d = \mu I_m$$

It is readily shown that the vector potential in region 2 is

$$A_z = \sum_{n=0}^{\infty} \frac{4a\mu I_m}{[(2n+1)\pi]^2} \frac{\cosh(2n+1)\pi d/a}{\sinh(2n+1)\frac{\pi b}{a}} \sin(2n+1)\frac{\pi x}{a} \cosh(2n+1)\frac{\pi(b-y)}{a}$$

therefore,

$$B_{y_2} = -\frac{\partial A_z}{\partial x} = -\sum_{n=0}^{\infty} \frac{4\mu I_m}{(2n+1)\pi} \frac{\cosh(2n+1)\pi d/a}{\sinh(2n+1)\pi b/a} \cos(2n+1)\frac{\pi x}{a} \cosh(2n+1)\pi\left(\frac{b-y}{a}\right)$$

$$B_{x_2} = \frac{\partial A_z}{\partial y} = -\sum_{n=0}^{\infty} \frac{4\mu I_m}{(2n+1)\pi} \frac{\cosh(2n+1)\pi d/a}{\sinh(2n+1)\pi b/a} \sin(2n+1)\frac{\pi x}{a} \sinh(2n+1)\pi\left(\frac{b-y}{a}\right)$$

If we let a/b≐ 11 and b/d≐ 3 and then scale Bx_2 to By_2, figure (6) illustrates the vector potential.

The average value of By for an area of a/2 (b-2d) is

$$Av\ By_2 = +\sum (-1)^{n+1} \frac{8\mu I_m a}{(b-2d)[(2n+1)\pi]^3} \frac{\cosh(2n+1)\frac{\pi d}{a} \quad \sinh(2n+1)\pi(b-2d)}{a \quad \sinh(2n+1)\pi\frac{b}{a}}$$

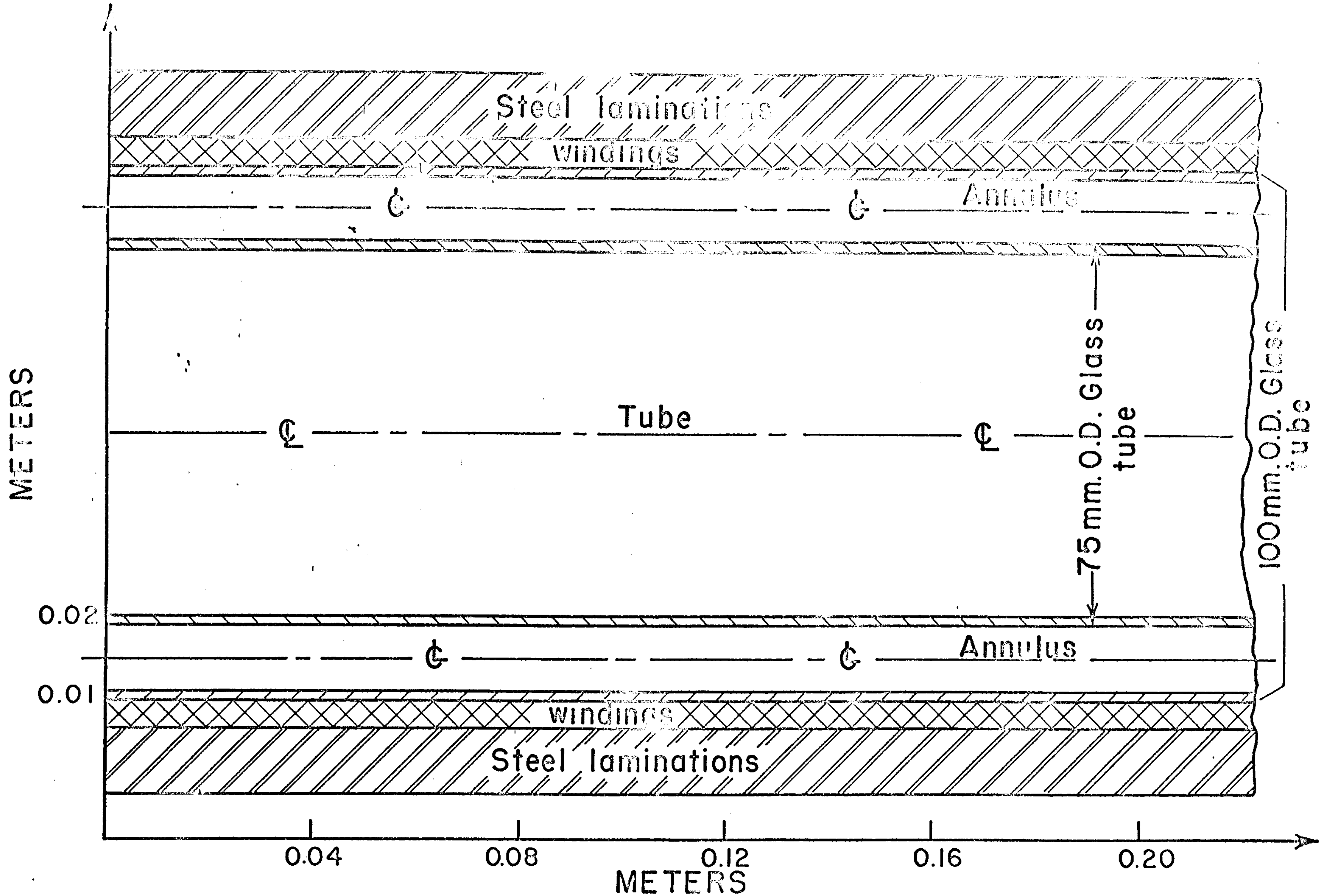


FIG 6. SECTION OF TRAVELING WAVE TUBE (One phase)

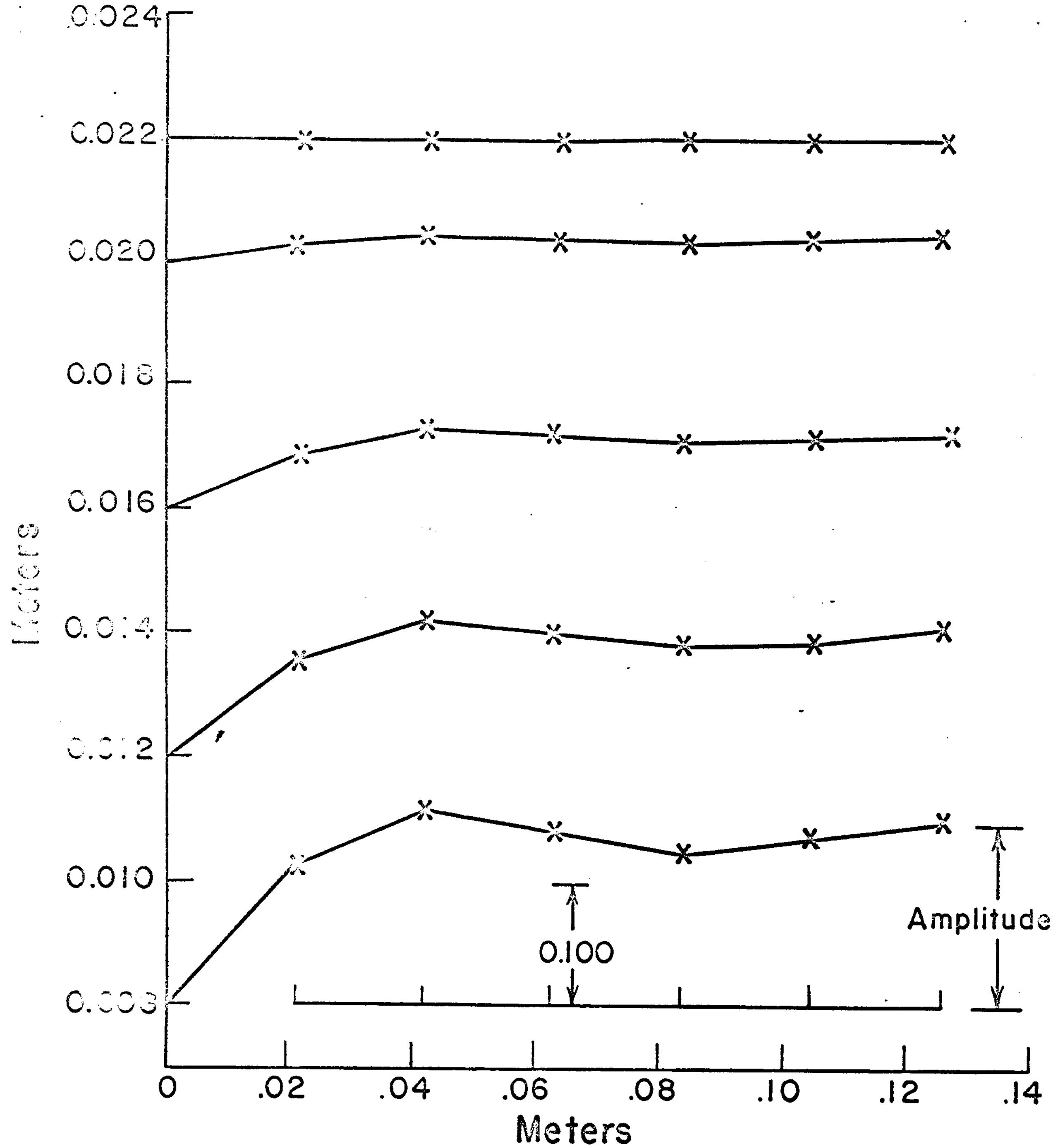


FIG 6a. AMPLITUDE of Bx_2 for SPECIFIC Y-DISTANCES vs PHASE DISTANCE

Curves show sum of first three harmonics

$$Bx_2 = -\sum_{n=0}^{\infty} \frac{4\mu I_m}{\pi(2n+1)} \; \frac{\cosh(2n+1)\frac{\pi d}{a}}{\sinh(2n+1)\frac{\pi b}{a}} \sinh(2n+1)\pi\left(\frac{b-y}{a}\right)\sin(2n+1)\frac{\pi x}{a}$$

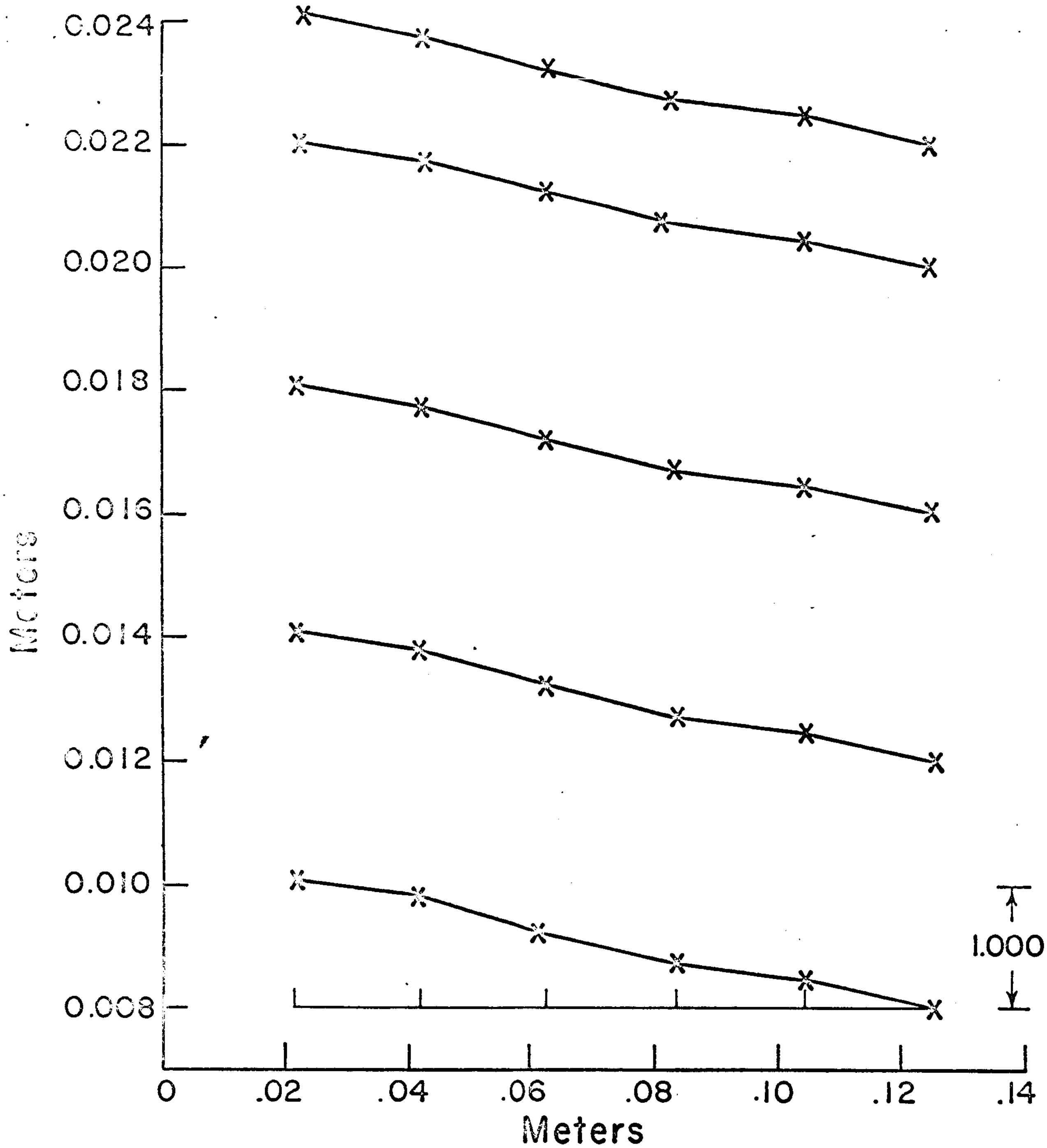


FIG 6b. AMPLITUDE of By_2 for SPECIFIC Y-DISTANCES vs PHASE DISTANCE

Curves show sum of first three harmonics

$$By_2 = -\sum_{n=0}^{\infty} \frac{4\mu I_m}{\pi(2n+1)} \frac{\cosh(2n+1)\frac{\pi d}{a}}{\sinh(2n+1)\frac{\pi b}{a}} \cos(2n+1)\frac{\pi x}{a} \cosh(2n+1)\pi\left(\frac{b-y}{a}\right)$$

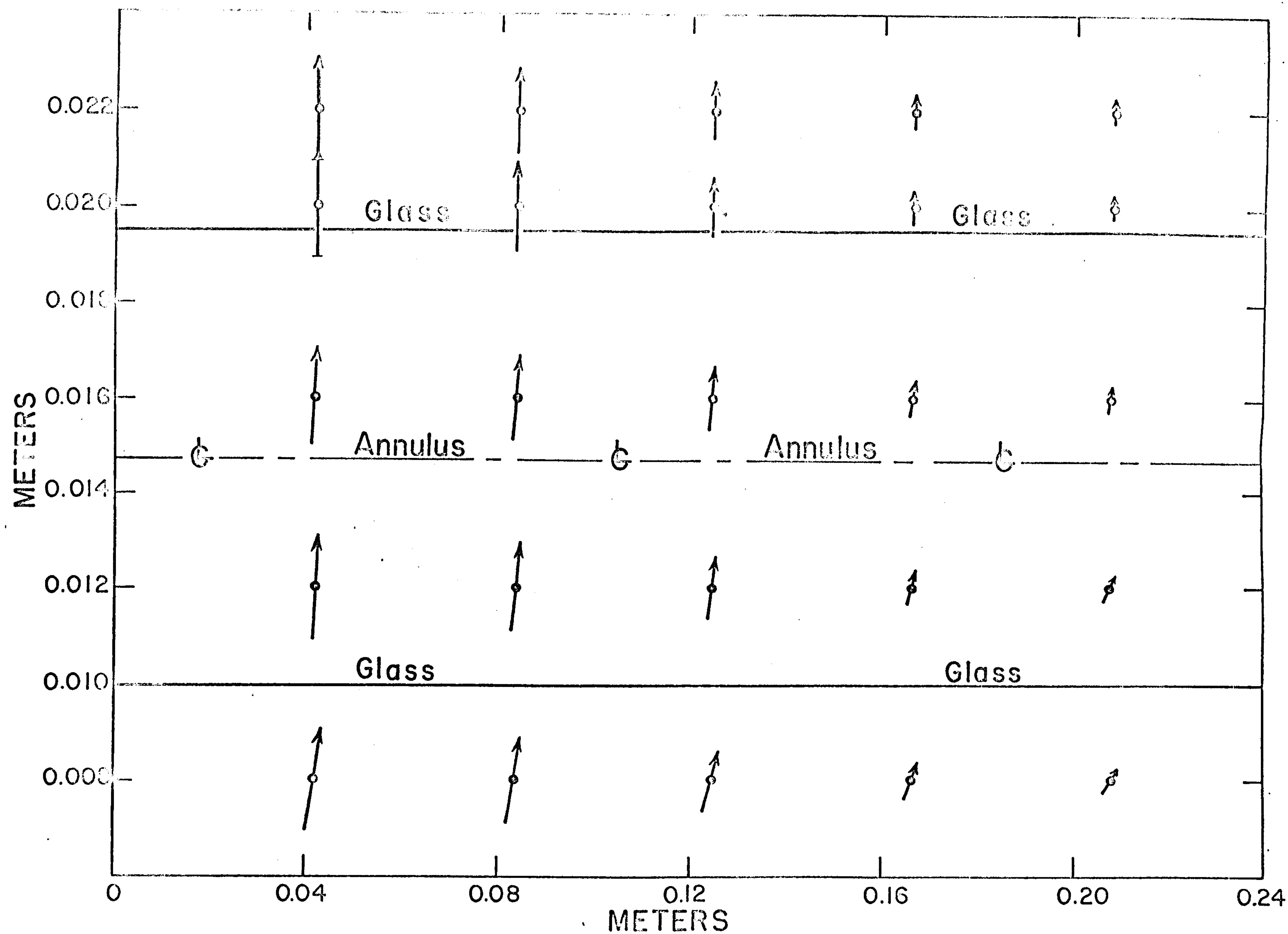


FIG 6c. COMPOSITE OF FIG. 6a. & 6b.

All that remains is to choose a convenient set of dimensions, introduce the maximum value of linear current density available and than calculate the magnitude of the largest radial B field attainable.

Experimental Program

As previously mentioned, the experimental work has proceeded along with the analytical work. The channel based on the one dimensional analysis is being constructed.

The solenoidal winding on the travelling wave tube has taps for both 3 phase and 4 phase connections. A preliminary investigation of one of the three phase windings revealed that

a) by capacitively tuning the circuit the maximum load could be increased by a factor of three.

b) when a core made of soft iron wire was inserted into the solenoid the inductive reactance went up by a factor of 100.

It is our intention to perform the necessary tests to determine the optimum balanced circuitry for the final installation which will contain both a laminated steel core and outer shell.

Considerable effort was expended in location core material with low eddy current losses and a realtive permeability above

500 for operation at 20 KC and magnetic field intensities of several hundred gauss. We chose a high Si content shell which measures 0.004" x 1/2" x 40.0" This appeared to be the best available material. Presently construction is nearly complete on the assembly of the 5000 laminations required with the appropriate spacers to assure a uniform radial stacking for both the core and the outer shell.

Power Supply

The power supply to be used for this experiment is rated at 20 KW, with a frequency of 20 KC and an output current of 90 amperes. This has been ordered for several months and delivery will be complete with the arrival of the motor starter promised about October 1. After this is received and connections made, a search of the magnetic field will be made to determine experimentally the magnetic field intensity in the channel.

For this purpose, a set of search coils have been designed that are mutually orthogonal and can be mounted on a probe to measure the strength of the magnetic field in the annular region of the travelling wave tube. The coils will sense radial and axial flux, and the induced voltage is read on an oscilloscope. The possibility of having any mutual inductant is removed if continuity is maintained in only one circuit at a time. Both coils can be read accurately to within one gauss.

Ion Generation and Diagnostics

Previously mentioned in the introduction was a glow discharge ionization device and power supply. The theory for long discharges vs. columns electrode size is well established and may be found in reference 5. However, for concentric cylindrical electrodes much less is known and for discharges with long electrodes compared to the gas discharge column very little seems to be published. For this reason some preliminary experimental work is now being conducted to determine volt ampere characteristics for the above geometrics, which have définite geometrical advantages for the possible production of ionized gases for the travelling wave device.

For this experimental work a power supply was needed which was designed and constructed to provide very low ripple d.c. from 0 to 600 volts and 2 amperes. This experiment is being performed now and sufficient information should be available for use within two or threeweeks of actual operation.

In connection with this experiment and for ion concentration determination within the travelling wave tube, Langmuir probes following the design and theory presented by French in reference 6 are currently being made.

The above describes the analytical and experimental research that has proceeded to date. It is intended to continue the analysis and experimental verification of the coaxial travelling wave device in the ensuing months as specified in the original contract. Below is a brief description of a possible future extension of this work.

1. Ion migration - coaxial conical configuration
2. Possible field geometry changes.
3. Continuous Flow Rail Gun
   (possible field due to rapid electron diffusion)

A Continuous Flow Rail Gun

The usual rail gun operation is affected by producing a discharge in a gas between two electrodes (rails) by means of an applied field. The back strapped electrodes than give rise to a B field perpendicular to the plane of the electrodes in a two dimensional model or a meridianal plane in case of a coaxial model. This field coupled to current density J caused by the flow of electrons relative to ions in the plasma between the electrodes give rise to a high magnetic pressure which accelerates the gas out of the tube. New gases introduced and the pulsing technique repeated. In the continuous model a glow discharge is considered over the entire length of the discharge tube. A continuous flow of gas is introduced and the device is operated in a steady state condition.

The pertinent equations for one fluid model are:

Continuity $$\frac{\partial \rho}{\partial t} + (\rho u_k)_{,k} = 0 \qquad (28)$$

Momentum $$\rho\left(\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k}\right) = e_{ijk} J_j B_k \qquad (29)$$

where:

ρ = the density

u_i = the ith component of velocity

x_k = the spacial coordinate

t = time

e_{ijk} = generalixed Kroneckerdelta

J_j = jth component of current density

B_k = kth component of magnetic field intensity

For a one dimensional, steady state condition the equations reduce to:

$$\frac{\partial(\rho u)}{\partial x} = 0 \qquad (30)$$

and

$$\rho u \frac{\partial u}{\partial x} = JB \qquad (31)$$

The directions of J and B are indicated in Figure 7.

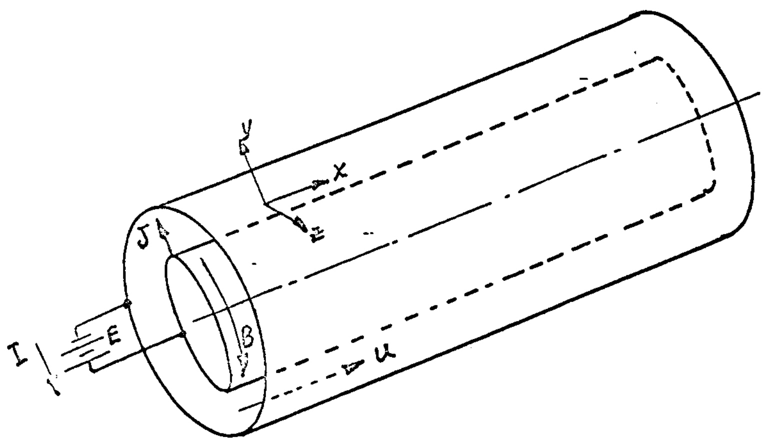


Figure 7

Coaxical Continuous Flow Rail Gun

In addition to equations (30) and (31), the Maxwell equations and Ohm's Law are:

$$\mu \bar{J} = \nabla \times \bar{B} \qquad (32)$$
$$-\frac{\partial \bar{B}}{\partial t} = \nabla \times \bar{E}$$

$$\bar{J} = \sigma [\bar{E} + \bar{v} \times \bar{B}] \qquad (33)$$

From the first of equations (32) $\mu J = -\frac{dB}{dx}$. (32a)

If this is inserted into (31)

$$\rho u \frac{du}{dx} = -\frac{1}{\mu} B \frac{dB}{dx} \qquad (34)$$

From (30) ρu = a constant $= \rho_0 u_0$

which substituted into (34) gives

$$\rho_0 u_0 \frac{du}{dx} + \frac{1}{\mu} \frac{d}{dx}\left(\frac{B^2}{2}\right) = 0$$

or $$\rho_0 u_0 u + \frac{1}{\mu} \frac{B^2}{2} = \rho_0 u_0^2 + \frac{1}{\mu} \frac{B_0^2}{2} \qquad (35)$$

Since B_0 (the magnetic field intensity) at X=0 is the largest in the tube and u_0 is assumed very low, the large part of the right hand side of (35) is probably contained in the second term. Furthermore B $\longrightarrow 0$ at the end of the channel, thus

u_f (An appropriate maximum velocity) $\approx \frac{1}{\rho_0 u_0 \mu} \frac{B_0^2}{2}$

Equation (35) is the expression for U or B in terms of the other. Combining equations (33) and (32a) for the steady state gives:

$$-\frac{1}{\mu}\frac{dB}{dx} = \sigma\left[E - uB\right] \tag{36}$$

Insertion of U from (35) gives:

$$-\frac{1}{\mu}\frac{dB}{dx} = \sigma\left[E + \left\{\frac{1}{\mu\rho_0 u_0}\left(\frac{B_0^2}{2} - \frac{B^2}{2}\right) + u_0\right\}B\right] \tag{37}$$

Since E is the applied field - assumed constant for this example say $E = E_0$

$$\frac{1}{\mu}\frac{dB}{dx} - \frac{\sigma B^3}{2\mu\rho_0 u_0} + \frac{\sigma B_0^2}{2\mu\rho u_0}B = -\sigma E_0 \tag{38}$$

This is Abel's differential equation for which solutions can be found readily.

In addition to the extension listed on Page 23 of the travelling wave device, we should like to consider the above model in a future project; to consider a multicomponent fluid model; charge exchange effects and possible E variation via segmented electrodes. There is also the effect of the rapid diffusion of electrons out of the tube in this nonequilibrian plasma model which are not accounted for above and which effect could be considerable, possibly even in excess of the magnetic pressure which _is_ included.

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